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The stress-intensity factor history for a half plane crack in a transversely isotropic solid due to impact point loading on the crack faces

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Abstract

Three-dimensional analysis is performed for a transversely isotropic solid containing a half plane crack subjected to suddenly applied concentrated point forces acting at a finite distance from the crack edge. The solution of this problem is treated as the superposition of two simpler problems. One considers the transient wave in an elastic half space generated by an impact point loading on the surface, the other problem is that which cancels out the surface displacement ahead of the crack edge induced by problem 1. A half space subjected to a distributed dislocation on the surface is constructed as the fundamental problem and solved by the use of integral transforms, the Wiener–Hopf technique and the Cagniard-de Hoop method. An exact expression is derived for the mode I stress-intensity factor as a function of time and position along the crack edge. Some features of the solution are discussed through numerical results. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Transverse isotropy; Half plane crack; Transient loading; Stress-intensity factor

1. Introduction

With the wide usage of macroscopically anisotropic construction materials such as geomaterials, crystals, and fiber-reinforced composites, great interest has been shown in the dynamic crack problems of anisotropic elasticity recently. The study of these problems is of particular importance to linear, elastic fracture mechanics to assess the initiation and growth of a developed macro-crack under dynamic loading conditions, and to nondestructive evaluation for detecting and characterizing the damaged state of materials. Elastodynamic analysis of a crack in an infinite transversely isotropic medium has been performed by Ohyoshi (1973) and Zhang and Gross (1993) for incident SH waves, by Dhawan (1982a,b) for incident P and SV waves. Diffraction of plane time-harmonic elastic waves has been investigated by Lobanov and Novichkov (1981) for an antiplane crack in an orthotropic half plane, and by Norris and Achenbach (1984) for a semi-infinite crack in an infinite transversely isotropic material. Studies for a periodic array of cracks in transversely isotropic solids have been presented by Zhang (1992) for the incident SH waves, and by Mandal and Ghosh (1994) for the incident P waves. Transient stress-intensity factors due to impact loading have been given by Kassir and Bandyopadhyay (1983) and Ang (1987) for an inplane crack in an infinite

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orthotropic or transversely isotropic solid, by Shindo et al. (1986, 1992) for a crack in an orthotropic strip, by Ang (1988) for an inplane crack in a transversely isotropic layered solid, and by Kuo (1984a,b) for an interface crack between orthotropic and fully anisotropic half planes. All of the above-mentioned references discuss two-dimensional crack problems. But perhaps, because of the mathematical complexity, three-dimensional crack problems of an anisotropic medium under dynamic loading have not yet received much attention. The interaction of time-harmonic elastic waves with a penny-shaped crack has been analyzed by Tsai (1982, 1988) who calculated the elastodynamic stress-intensity factors, by Kundu and Bostrom (1991, 1992) who computed the scattered far-field. The three-dimensional analysis of cracks in the layered transversely isotropic media has been treated by Lin and Keer (1989). The ultrasonic crack detection in anisotropic materials has been investigated by Mattsson et al. (1997). Closed form solutions for a half plane crack in a transversely isotropic material due to both impact and moving loads have been obtained by Xiaohua et al. (1999, 2000).

In the present paper, three-dimensional analysis is performed for a transversely isotropic solid containing a half plane crack subjected to suddenly applied concentrated point forces acting at a finite distance from the crack edge. Different from those discussed by Xiaohua et al. (1999, 2000), this problem has a characteristic length in the loading function. Aside from being of importance in the field of dynamic fracture mechanics (Freund, 1990), its solution is of practical interest for engineering applications since the model of a half plane crack may be applied to any case for which we are interested in the stress distribution in a cracked body, with the distance from the loading to the crack edge being small as compared to the crack edge curvature, while the solution serves as a fundamental one. However, due to the existence of the characteristic length, the transform methods together with the Wiener–Hopf technique used by Xiaohua et al. (1999, 2000) cannot be directly applied. Here, an alternative methodology is developed. We treat such a solution as the superposition of two simpler problems. One considers the transient wave in an elastic half space generated by an impact point loading on the surface, the other problem is that which cancels out the surface displacement ahead of the crack edge induced by problem 1. A half space subjected to a distributed dislocation on the surface is constructed as the fundamental problem and solved by the use of integral transforms, the Wiener–Hopf technique and the Cagniard-de Hoop method. An exact expression is derived for the mode I stress intensity factor as a function of time and position along the crack edge. Some features of the solution are discussed through numerical results.

2. Basic formulas

Consider a transversely isotropic, linear elastic solid containing a half plane crack depicted in Fig. 1. The solid is initially stress free and at rest. A right-handed rectangular coordinate system is introduced such that the y-axis coincides with the crack edge, and the half plane crack occupies the area $z = 0$ and $x < 0$. At time $t = 0$, an opposed pair of point loads suddenly begins to act on the crack faces at a point at a finite distance l from the crack edge, resulting in a three-dimensional stress-wave field in the solid.

Let $u_x(x, y, z, t)$, $u_y(x, y, z, t)$ and $u_z(x, y, z, t)$ denote the relevant displacement components in the x, y and z directions, respectively, then the stresses in the solid can be expressed by the relations

$$\sigma_{xx} = c_1 \frac{\partial u_x}{\partial x} + c_2 \frac{\partial u_y}{\partial y} + c_3 \frac{\partial u_z}{\partial z}, \tag{1a}$$

$$\sigma_{yy} = c_2 \frac{\partial u_x}{\partial x} + c_1 \frac{\partial u_y}{\partial y} + c_3 \frac{\partial u_z}{\partial z}, \tag{1b}$$

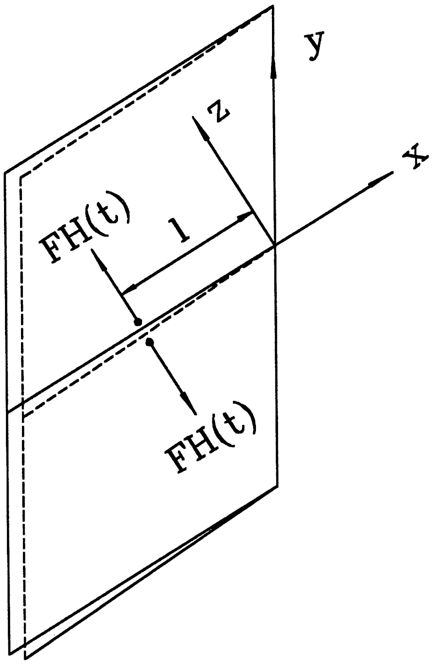

Fig. 1. Geometrical configuration of the elastic solid.

$$\sigma_{zz} = c_3\frac{\partial u_x}{\partial x} + c_3\frac{\partial u_y}{\partial y} + c_4\frac{\partial u_z}{\partial z}, \tag{1c}$$

$$\sigma_{yz} = c_5\left[\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}\right], \tag{1d}$$

$$\sigma_{xz} = c_5\left[\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right], \tag{1e}$$

$$\sigma_{xy} = \frac{1}{2}(c_1 - c_2)\left[\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right], \tag{1f}$$

where c_k $(k = 1,2,3,4,5)$ are material constants.

Equations of motion for the problem are

$$\sigma_{ij,j} = \rho\ddot{u}_i \quad (i = x, y, z), \tag{2}$$

where ρ is the material density.

For a transversely isotropic material, it proves convenient to introduce scalar potentials $\phi(x,y,z,t)$, $\psi(x,y,z,t)$ and $\theta(x,y,z,t)$, so the displacement components can be represented as

$$u_x = \frac{\partial\phi}{\partial x} + \frac{\partial\psi}{\partial y}, \tag{3a}$$

$$u_y = \frac{\partial\phi}{\partial y} - \frac{\partial\psi}{\partial x}, \tag{3b}$$

$$u_z = \frac{\partial \theta}{\partial z} \tag{3c}$$

Substitution of the above equations into Eqs. (1a)–(1f) and (2) gives after some manipulation

$$a_4 \nabla^2 \psi + a_5 \frac{\partial^2 \psi}{\partial z^2} = \frac{\partial^2 \psi}{\partial t^2}, \tag{4a}$$

$$a_3 \nabla^2 \phi + a_5 \nabla^2 \theta + a_2 \frac{\partial^2 \theta}{\partial z^2} = \frac{\partial^2 \theta}{\partial t^2}, \tag{4b}$$

$$a_1 \nabla^2 \phi + a_5 \frac{\partial^2 \phi}{\partial z^2} + a_3 \frac{\partial^2 \theta}{\partial z^2} = \frac{\partial^2 \phi}{\partial t^2}, \tag{4c}$$

where $\nabla^2 = (\partial^2/\partial x^2) + (\partial^2/\partial y^2)$, and the five constants $a_1 = c_1/\rho$, $a_2 = c_4/\rho$, $a_3 = (c_5 + c_3)/\rho$, $a_4 = (c_1 - c_2)/2\rho$, $a_5 = c_5/\rho$.

Due to the symmetry with respect to the plane $z = 0$, only the region $z \geqslant 0$ need be considered. The boundary conditions for $z = 0$ are

$$\sigma_{zz}(x, y, 0, t) = \sigma_-(x, y, t) + \sigma_+(x, y, t), \tag{5a}$$

$$\sigma_{xz}(x, y, 0, t) = 0, \tag{5b}$$

$$\sigma_{yz}(x, y, 0, t) = 0, \tag{5c}$$

$$u_z(x, y, 0, t) = u_-(x, y, t), \tag{5d}$$

for $-\infty < x, y < +\infty$, $t \geqslant 0$ and

$$\sigma_-(x, y, t) = -F\delta(x + l)\delta(y)H(t). \tag{6}$$

In Eqs. (5a)–(5d) and (6), F is the intensity of loads, $H(\cdot)$ is the Heaviside function and $\delta(\cdot)$ is the Dirac delta function. The function $\sigma_+(x, y, t)$ represents the unknown component of stress $\sigma_{zz}(x, y, 0, t)$ on $x > 0$, and $\sigma_+(x, y, t) \equiv 0$ for $x < 0$. The function $u_-(x, y, t)$ represents the unknown component of displacement $u_z(x, y, 0, t)$ on the crack faces for $x < 0$ and $u_-(x, y, t) \equiv 0$ in the half range $x > 0$.

The initial conditions are expressed in terms of the potentials as

$$\phi(x, y, z, 0) = \psi(x, y, z, 0) = \theta(x, y, z, 0) = 0, \tag{7a}$$

$$\frac{\partial \phi(x, y, z, 0)}{\partial t} = \frac{\partial \psi(x, y, z, 0)}{\partial t} = \frac{\partial \theta(x, y, z, 0)}{\partial t} = 0. \tag{7b}$$

Within the framework of linear elasticity, the solution of the formulated problem can be obtained by linear superposition of the solutions of two simpler problems. They are as follows:

Problem 1: Solve Eq. (4) with the nonmixed boundary conditions

$$\sigma^{\mathrm{I}}_{zz}(x, y, 0, t) = \sigma_-(x, y, t), \tag{8a}$$

$$\sigma^{\mathrm{I}}_{xz}(x, y, 0, t) = 0, \tag{8b}$$

$$\sigma^{\mathrm{I}}_{yz}(x, y, 0, t) = 0. \tag{8c}$$

Problem 2: Find the solution of Eq. (4) subjected to the mixed boundary conditions

$$\sigma^{\mathrm{II}}_{zz}(x, y, 0, t) = \sigma_+(x, y, t), \tag{9a}$$

$$\sigma_{xz}^{\mathrm{II}}(x,y,0,t)=0, \tag{9b}$$

$$\sigma_{yz}^{\mathrm{II}}(x,y,0,t)=0, \tag{9c}$$

$$u_z^{\mathrm{II}}(x,y,0,t)=u_-^{\mathrm{II}}(x,y,t)-u_+^{\mathrm{I}}(x,y,t). \tag{9d}$$

Solutions of both problems must also satisfy the initial conditions (7a) and (7b).

One can easily find that problem 1 considers the transient response of an elastic half space due to the application of an impact point traction at $x=-l$, $y=z=0$. This problem is usually called Lamb's problem, and its solution can be obtained by using integral transforms. For the sake of simplicity, the detailed calculation is omitted here and may be found in the work by Xiaohua (1999). A similar procedure can also be found in Eringen and Suhubi (1975) for the case of an isotropic material, the difference being that the integration contour for the inversion of transforms is modified to include all the branch cuts of the integrand. The final result for the surface displacement in the z direction takes the following form:

$$u_z^{\mathrm{I}}(x,y,0,t)=-\frac{F}{\pi^2\rho}\int_{p_1/p_2}^{\infty}\frac{H(\tau_0-v)}{p_2r(\tau_0^2-v^2)^{1/2}}f(v)\,\mathrm{d}v, \tag{10}$$

where

$$p_1^2=a_1^{-1},\quad p_2^2=a_5^{-1}, \tag{11}$$

$$r=\sqrt{(x+l)^2+y^2}, \tag{12}$$

$$\tau_0=\frac{t}{p_2r}. \tag{13}$$

In addition, when $p_1/p_2<v<1$,

$$f(v)=\frac{vQ_1Q_2}{\left[(1-2v^2)^2+a_5Pv^2+a_5^2Q\right]^2+(4v^2+a_5P)^2(v^2-a_5/a_1)(1-v^2)}, \tag{14}$$

$$P=\frac{4(\sqrt{a_1a_2}-a_2)}{a_1a_2-(a_3-a_5)^2}, \tag{15}$$

$$Q=\frac{P}{\sqrt{a_1a_2}}+\frac{a_2(a_2-a_1)+(a_3+a_5-a_2)(a_3+a_2-3a_5)}{a_5^2[a_1a_2-(a_3-a_5)^2]}, \tag{16}$$

$$Q_1=\frac{4\sqrt{a_1a_2}}{a_1a_2-(a_3-a_5)^2}, \tag{17}$$

$$Q_2=\left(v^2-\frac{a_5}{a_1}\right)(1+a_5P+a_5^2Q)\beta_2+\left[1+4v^2\left(\frac{a_5}{a_1}-1\right)+\frac{Pa_5^2}{a_1}+a_5^2Q\right]\beta_1\sqrt{v^2-\frac{a_5}{a_1}}\sqrt{1-v^2}, \tag{18}$$

$$\beta_1=\left\{\left[\left(\frac{Lv^2+a_2a_5+a_5^2}{2a_2a_5^2}\right)^2+\frac{a_1}{a_2a_5^2}\left(v^2-\frac{a_5}{a_1}\right)(1-v^2)\right]^{1/2}+\frac{Lv^2+a_2a_5+a_5^2}{2a_2a_5^2}\right\}^{1/2}, \tag{19}$$

$$\beta_2 = \left\{ \left[\left(\frac{Lv^2 + a_2a_5 + a_5^2}{2a_2a_5^2} \right)^2 + \frac{a_1}{a_2a_5^2} \left(v^2 - \frac{a_5}{a_1} \right) (1 - v^2) \right]^{1/2} - \frac{Lv^2 + a_2a_5 + a_5^2}{2a_2a_5^2} \right\}^{1/2}, \tag{20}$$

$$L = a_3^2 - a_5^2 - a_1a_2. \tag{21}$$

When $v > 1$,

$$f(v) = \frac{vQ_3\sqrt{v^2 - a_5/a_1}}{(1 - 2v^2)^2 + a_5Pv^2 + a_5^2Q - (4v^2 + a_5P)\sqrt{v^2 - a_5/a_1}\sqrt{v^2 - 1}}, \tag{22}$$

$$Q_3 = \frac{4\sqrt{a_1a_2}(a_1 - a_5)}{a_1\left[a_1a_2 - (a_3 - a_5)^2\right]} \frac{\beta_3 + \beta_4}{\sqrt{v^2 - a_5/a_1} + \sqrt{v^2 - 1}}, \tag{23}$$

$$\beta_3 = \left\{ -\left[\left(\frac{Lv^2 + a_2a_5 + a_5^2}{2a_2a_5^2} \right)^2 - \frac{a_1}{a_2a_5^2} \left(v^2 - \frac{a_5}{a_1} \right) (v^2 - 1) \right]^{1/2} - \frac{Lv^2 + a_2a_5 + a_5^2}{2a_2a_5^2} \right\}^{1/2}, \tag{24}$$

$$\beta_4 = \left\{ \left[\left(\frac{Lv^2 + a_2a_5 + a_5^2}{2a_2a_5^2} \right)^2 - \frac{a_1}{a_2a_5^2} \left(v^2 - \frac{a_5}{a_1} \right) (v^2 - 1) \right]^{1/2} - \frac{Lv^2 + a_2a_5 + a_5^2}{2a_2a_5^2} \right\}^{1/2}. \tag{25}$$

Thus, we have

$$u_+^{\mathrm{I}}(x, y, t) = u_z^{\mathrm{I}}(x, y, 0, t)H(x). \tag{26}$$

3. Required fundamental solution

As the first step of solving problem 2, a fundamental problem is constructed. The problem can be viewed as a half-space problem with the material occupying the region $z \geqslant 0$, and is subjected to the following mixed boundary conditions for $z = 0$:

$$\sigma_{zz}^{\mathrm{F}}(x, y, 0, t) = \sigma_+^{\mathrm{F}}(x, y, t), \tag{27a}$$

$$\sigma_{xz}^{\mathrm{F}}(x, y, 0, t) = 0, \tag{27b}$$

$$\sigma_{yz}^{\mathrm{F}}(x, y, 0, t) = 0, \tag{27c}$$

$$u_z^{\mathrm{F}}(x, y, 0, t) = u_-^{\mathrm{F}}(x, y, t) - \frac{H(\tau_0 - v)H(x)}{p_2r(\tau_0^2 - v^2)^{1/2}}. \tag{27d}$$

This fundamental problem can be solved by using transform methods and the Wiener–Hopf technique. Initially, a one-sided Laplace transform over time is applied to the partial differential Eqs. (4a)–(4c), taking into account the initial conditions (7a) and (7b). The transformed function is denoted by a superposed hat, for example,

$$\widehat{\phi}(x, y, z, s) = \int_0^\infty \phi(x, y, z, t)\mathrm{e}^{-st}\mathrm{d}t, \tag{28}$$

where the complex number s has a positive real part. Thereafter, a two-sided Laplace transform is introduced over the y-coordinate. The complex transform parameter is $s\xi$, and the transformed function is denoted by a bar, i.e.

$$\overline{\phi}(x,\xi,z,s)=\int_{-\infty}^{+\infty}\widehat{\phi}(x,y,z,s)\mathrm{e}^{-s\xi y}\,\mathrm{d}y. \tag{29}$$

Finally, a two-sided Laplace transform is used to suppress the dependence on x. The complex transform parameter is $s\eta$, and the transformed function is denoted as

$$\phi^*(\eta,\xi,z,s)=\int_{-\infty}^{+\infty}\overline{\phi}(x,\xi,z,s)\mathrm{e}^{-s\eta x}\,\mathrm{d}x. \tag{30}$$

The partial differential Equations (4a)–(4c) are reduced to

$$-a_4s^2\mu_3^2\psi^*+a_5\frac{\mathrm{d}^2\psi^*}{\mathrm{d}z^2}=0, \tag{31a}$$

$$a_3s^2(\eta^2+\xi^2)\phi^*-a_5s^2\mu_2^2\theta^*+a_2\frac{\mathrm{d}^2\theta^*}{\mathrm{d}z^2}=0, \tag{31b}$$

$$-a_1s^2\mu_1^2\phi^*+a_5\frac{\mathrm{d}^2\phi^*}{\mathrm{d}z^2}+a_3\frac{\mathrm{d}^2\theta^*}{\mathrm{d}z^2}=0, \tag{31c}$$

where

$$\mu_1(\eta,\xi)=(p_1^2-\eta^2-\xi^2)^{1/2}, \tag{32}$$

$$\mu_2(\eta,\xi)=(p_2^2-\eta^2-\xi^2)^{1/2}, \tag{33}$$

$$\mu_3(\eta,\xi)=(p_3^2-\eta^2-\xi^2)^{1/2}, \tag{34}$$

$$p_3^2=a_4^{-1}. \tag{35}$$

The bounded solutions to Eqs. (31a)–(31c) as $z\to\infty$ may be written in the form

$$\phi^*=A\mathrm{e}^{-s\lambda_1z}+B\mathrm{e}^{-s\lambda_2z}, \tag{36a}$$

$$\theta^*=\frac{a_1\mu_1^2-a_5\lambda_1^2}{a_3\lambda_1^2}A\mathrm{e}^{-s\lambda_1z}+\frac{a_1\mu_1^2-a_5\lambda_2^2}{a_3\lambda_2^2}B\mathrm{e}^{-s\lambda_2z}, \tag{36b}$$

$$\psi^*=C\mathrm{e}^{-s\lambda_3z}, \tag{36c}$$

where A, B, C are arbitrary functions of ξ and η, and

$$\lambda_{1,2}^2=\frac{L(\eta^2+\xi^2)+a_2+a_5}{2a_2a_5}\pm\sqrt{\left[\frac{L(\eta^2+\xi^2)+a_2+a_5}{2a_2a_5}\right]^2-\frac{a_1}{a_2}\mu_1^2\mu_2^2}, \tag{37}$$

$$\lambda_3=\sqrt{\frac{a_4}{a_5}}\mu_3. \tag{38}$$

The complex η plane is cut along $\sqrt{p_1^2-\xi^2}<|\mathrm{Re}(\eta)|<\infty$, $\mathrm{Im}(\eta)=0$. So that $\mathrm{Re}(\mu_1)\geqslant 0$ in the entire cut η plane for each value of η, and likewise for $\mathrm{Re}(\mu_2,\mu_3,\lambda_1,\lambda_2)\geqslant 0$.

Now, if we make use of the known integrals (Erdelyi, 1954; Freund, 1990)

$$\int_0^{\infty} \frac{H(\tau_0 - v)}{p_2 r(\tau_0^2 - v^2)^{1/2}} \exp(-st)\,\mathrm{d}t = K_0(p_2 srv), \tag{39}$$

$$\int_{-\infty}^{+\infty} K_0(p_2 srv) \exp(-s\xi y)\,\mathrm{d}y = \frac{\pi}{s\lambda} \exp[-s(x+l)\lambda], \tag{40}$$

where $K_0(p_2 srv)$ is the modified Bessel function of the second kind and

$$\lambda = \lambda(\xi) = \sqrt{p_2^2 v^2 - \xi^2}, \tag{41}$$

the Laplace transformation of boundary conditions (27a)–(27d), together with the transformed stresses and displacements, will lead to

$$\rho s^3 \left[(a_3 - a_5)(\eta^2 + \xi^2) + \frac{a_2}{a_3}(a_1\mu_1^2 - a_5\lambda_1^2) \right] A + \rho s^3 \left[(a_3 - a_5)(\eta^2 + \xi^2) + \frac{a_2}{a_3}(a_1\mu_1^2 - a_5\lambda_2^2) \right] B = \Sigma_+, \tag{42a}$$

$$\left(\frac{a_1\mu_1^2 - a_5\lambda_1^2}{a_3\lambda_1} + \lambda_1 \right) \xi A + \left(\frac{a_1\mu_1^2 - a_5\lambda_2^2}{a_3\lambda_2} + \lambda_2 \right) \xi B - \eta\lambda_3 C = 0, \tag{42b}$$

$$\left(\frac{a_1\mu_1^2 - a_5\lambda_1^2}{a_3\lambda_1} + \lambda_1 \right) \eta A + \left(\frac{a_1\mu_1^2 - a_5\lambda_2^2}{a_3\lambda_2} + \lambda_2 \right) \eta B + \xi\lambda_3 C = 0, \tag{42c}$$

$$-s^3 \left(\frac{a_1\mu_1^2 - a_5\lambda_1^2}{a_3\lambda_1} A + \frac{a_1\mu_1^2 - a_5\lambda_2^2}{a_3\lambda_2} B \right) = U_- - \frac{\pi}{\lambda(\lambda + \eta)} \exp(-sl\lambda). \tag{42d}$$

If A, B, C are eliminated from the above equations, we will obtain

$$-\frac{\rho R(\eta, \xi)}{\mu_1(\eta, \xi)} \left[U_- - \frac{\pi}{\lambda(\lambda + \eta)} \exp(-sl\lambda) \right] = \Sigma_+ \tag{43}$$

in which

$$\Sigma_+ = s \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \widehat{\sigma}_+^{\mathrm{F}}(x, y, s) \exp[-s(\xi y + \eta x)]\,\mathrm{d}y\,\mathrm{d}x, \tag{44}$$

$$U_- = s^2 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \widehat{u}_-^{\mathrm{F}}(x, y, s) \exp[-s(\xi y + \eta x)]\,\mathrm{d}y\,\mathrm{d}x, \tag{45}$$

$$R(\eta, \xi) = \frac{\left\{ [(a_3 - a_5)^2 - a_1 a_2](\eta^2 + \xi^2) + a_2 \right\} \mu_2 + \sqrt{a_1 a_2}\,\mu_1}{\sqrt{a_1 a_2}(\lambda_1 + \lambda_2)}. \tag{46}$$

Eq. (43) is of the type that can be solved by Wiener–Hopf technique. So, we may determine the two unknown functions Σ_+ and U_- with a single equation. The Wiener–Hopf procedure requires that the mixed functions in Eq. (43) must be factored into the product of sectionally analytic functions. For this purpose, the function $R(\eta, \xi)$ may be written in the form

$$R(\eta,\xi)=\frac{a_1a_5[a_1a_2-(a_3-a_5)^2](\mu_1+\mu_2)}{4\sqrt{a_1a_2}(a_1-a_5)(\lambda_1+\lambda_2)}\left\{4(\eta^2+\xi^2)\mu_1\mu_2+\left(\frac{1}{a_5}-2\eta^2-2\xi^2\right)^2+P[(\eta^2+\xi^2)+\mu_1\mu_2]+Q\right\}. \tag{47}$$

It is found that through appropriate manipulation $R(\eta,\xi)$ can be transformed into a form identical to Eq. (3.27) of the paper by Buchwald (1961), in which it is proved that $R(\eta,\xi)=0$ has only two roots $\eta=\pm\sqrt{c^2-\xi^2}$ along the real axis. Here $c=c_r^{-1}$, c_r is the Rayleigh wave speed for a transversely isotropic solid.

Now, consider a new function $S(\eta,\xi)$ defined by

$$S(\eta,\xi)=\frac{R(\eta,\xi)}{k(c^2-\eta^2-\xi^2)}, \tag{48}$$

then

$$S(\eta,\xi)=S_1(\eta,\xi)S_2(\eta,\xi), \tag{49}$$

$$S_1(\eta,\xi)=\frac{a_1a_5}{2(a_1-a_5)}\frac{4(\eta^2+\xi^2)\mu_1\mu_2+\left(\frac{1}{a_5}-2\eta^2-2\xi^2\right)^2+P[(\eta^2+\xi^2)+\mu_1\mu_2]+Q}{c^2-\eta^2-\xi^2}, \tag{50}$$

$$S_2(\eta,\xi)=\frac{a_1a_2-(a_3-a_5)^2}{2k\sqrt{a_1a_2}}\frac{\mu_1+\mu_2}{\lambda_1+\lambda_2}, \tag{51}$$

$$k=\frac{(2a_2a_5)^{1/2}}{\sqrt{a_1a_2}}\frac{a_1a_2-(a_3-a_5)^2}{\left[-(L^2-4a_1a_2a_5^2)^{1/2}-L\right]^{1/2}+\left[(L^2-4a_1a_2a_5^2)^{1/2}-L\right]^{1/2}}. \tag{52}$$

The function $S_1(\eta,\xi)$ has no poles or zeros in the complex η plane, the only singularities being the branch points of the functions $\mu_1(\eta,\xi)$ and $\mu_2(\eta,\xi)$. In the entire cut η plane, $S_1(\eta,\xi)$ is analytic. When $|\eta|\to\infty$, $S_1(\eta,\xi)\to 1$. According to Cauchy's integral theorem, $S_1(\eta,\xi)$ can be decomposed into

$$S_1(\eta,\xi)=S_1^+(\eta,\xi)S_1^-(\eta,\xi) \tag{53}$$

with

$$S_1^\pm(\eta,\xi)=\exp\left\{-\frac{1}{\pi}\int_{p_1}^{p_2}tg^{-1}\left[\frac{(4\varsigma^2+P)\sqrt{p_2^2-\varsigma^2}\sqrt{\varsigma^2-p_1^2}}{(p_2^2-2\varsigma^2)^2+P\varsigma^2+Q}\right]\frac{\varsigma\,\mathrm{d}\varsigma}{\sqrt{\varsigma^2-\xi^2}\left(\sqrt{\varsigma^2-\xi^2}\pm\eta\right)}\right\}. \tag{54}$$

The functions $S_1^+(\eta,\xi)$ and $S_1^-(\eta,\xi)$ are analytic and nonzero in the half planes $\mathrm{Re}(\eta)>-\sqrt{p_1^2-\xi^2}$ and $\mathrm{Re}(\eta)<\sqrt{p_1^2-\xi^2}$, respectively.

With reference to Eq. (49), the function $S_2(\eta,\xi)$ remains to be split. The singularities of $S_2(\eta,\xi)$ in the complex η plane are the branch points of the functions $\lambda_1(\eta,\xi)$, $\lambda_2(\eta,\xi)$, $\mu_1(\eta,\xi)$ and $\mu_2(\eta,\xi)$. The functions $\lambda_1(\eta,\xi)$ and $\lambda_2(\eta,\xi)$ possess two kinds of branch points. The first kind are the points given by

$$\lambda_1(\eta,\xi)=0,\qquad \lambda_2(\eta,\xi)=0. \tag{55}$$

The substitution of Eq. (37) into Eq. (55) leads to the following branch points:

$$\eta = \pm\sqrt{p_1^2 - \xi^2}, \qquad \eta = \pm\sqrt{p_2^2 - \xi^2}, \tag{56}$$

which are also the branch points of $\mu_1(\eta, \xi)$ and $\mu_2(\eta, \xi)$. The second kind of branch points are where $\lambda_2^2(\eta, \xi) - \lambda_1^2(\eta, \xi)$ is zero. Such points will occur in pairs. Between the two points of a given pair, we may define a branch cut such that $\lambda_1(\eta, \xi) + \lambda_2(\eta, \xi)$ is continuous across the cut, while $\lambda_1(\eta, \xi)$ and $\lambda_2(\eta, \xi)$ are each discontinuous. Therefore, these cuts give no contribution to the analytic factorization. So, we obtain via the use of Cauchy's integral theorem

$$S_2(\eta, \xi) = S_2^+(\eta, \xi) S_2^-(\eta, \xi), \tag{57}$$

$$S_2^{\pm}(\eta, \xi) = \exp\left\{ -\frac{1}{\pi}\int_{p_1}^{p_2} tg^{-1}\left[\frac{\beta_5\sqrt{\varsigma^2 - p_1^2} - \beta_6\sqrt{p_2^2 - \varsigma^2}}{\beta_5\sqrt{p_2^2 - \varsigma^2} + \beta_6\sqrt{\varsigma^2 - p_1^2}}\right] \frac{\varsigma\, d\varsigma}{\sqrt{\varsigma^2 - \xi^2}\left(\sqrt{\varsigma^2 - \xi^2} \pm \eta\right)} \right\}, \tag{58}$$

where

$$\beta_5 = \left\{ \left[\left(\frac{L\varsigma^2 + a_2 + a_5}{2a_2 a_5} \right)^2 + \frac{a_1}{a_2}(\varsigma^2 - p_1^2)(p_2^2 - \varsigma^2) \right]^{1/2} + \frac{L\varsigma^2 + a_2 + a_5}{2a_2 a_5} \right\}^{1/2}, \tag{59}$$

$$\beta_6 = \left\{ \left[\left(\frac{L\varsigma^2 + a_2 + a_5}{2a_2 a_5} \right)^2 + \frac{a_1}{a_2}(\varsigma^2 - p_1^2)(p_2^2 - \varsigma^2) \right]^{1/2} - \frac{L\varsigma^2 + a_2 + a_5}{2a_2 a_5} \right\}^{1/2}. \tag{60}$$

The functions $S_2^+(\eta, \xi)$ and $S_2^-(\eta, \xi)$ are analytic and nonzero in the half planes $\mathrm{Re}(\eta) > -\sqrt{p_1^2 - \xi^2}$ and $\mathrm{Re}(\eta) < \sqrt{p_1^2 - \xi^2}$, respectively.

Thus, we have

$$S(\eta, \xi) = S_+(\eta, \xi) S_-(\eta, \xi) \tag{61}$$

with

$$S_{\pm}(\eta, \xi) = S_1^{\pm}(\eta, \xi) S_2^{\pm}(\eta, \xi). \tag{62}$$

We also have

$$\sqrt{p_1^2 - \xi^2 - \eta^2} = \sqrt{\sqrt{p_1^2 - \xi^2} + \eta}\sqrt{\sqrt{p_1^2 - \xi^2} - \eta} \tag{63}$$

and

$$c^2 - \xi^2 - \eta^2 = \left(\sqrt{c^2 - \xi^2} + \eta\right)\left(\sqrt{c^2 - \xi^2} - \eta\right). \tag{64}$$

Let

$$F_{\pm}(\eta, \xi) = \frac{\left(\sqrt{p_1^2 - \xi^2} \pm \eta\right)^{1/2}}{\left(\sqrt{c^2 - \xi^2} \pm \eta\right) S_{\pm}(\eta, \xi)}, \tag{65}$$

then Eq. (43) becomes

$$-\frac{\rho k}{F_-(\eta,\xi)}\left[U_- - \frac{\pi}{\lambda(\lambda+\eta)}\exp(-sl\lambda)\right] = F_+(\eta,\xi)\Sigma_+. \tag{66}$$

The only singularity of the mixed function in Eq. (66) in the left-half plane is a simple pole at $\eta = -\lambda$. This singularity can be removed by requiring the residue to be zero, so we obtain

$$-\frac{\rho k}{F_-(\eta,\xi)}U_- + \frac{\pi\rho k\exp(-sl\lambda)}{\lambda(\lambda+\eta)}\left[\frac{1}{F_-(\eta,\xi)} - \frac{1}{F_+(\lambda,\xi)}\right] = F_+(\eta,\xi)\Sigma_+ - \frac{\pi\rho k\exp(-sl\lambda)}{\lambda(\lambda+\eta)F_+(\lambda,\xi)}. \tag{67}$$

The right-hand side of Eq. (67) is analytic for $\mathrm{Re}(\eta) > -\sqrt{p_1^2-\xi^2}$, and the left-hand side is analytic for $\mathrm{Re}(\eta) < \sqrt{p_1^2-\xi^2}$. Hence, by analytic continuation, each side of Eq. (67) represents the same entire function $E(\eta,\xi,s)$. According to Liouville's theorem, a bounded entire function is a constant. In this case, $E(\eta,\xi,s)$ is bounded in the finite plane and $E(\eta,\xi,s)\to 0$ as $|\eta|\to\infty$. Thus, $E(\eta,\xi,s)\equiv 0$, and we have

$$\Sigma_+ = \frac{\pi\rho k\exp(-sl\lambda)}{\lambda(\lambda+\eta)F_+(\lambda,\xi)F_+(\eta,\xi)}, \tag{68}$$

$$U_- = \frac{\pi\exp(-sl\lambda)}{\lambda(\lambda+\eta)}\left[1 - \frac{F_-(\eta,\xi)}{F_+(\lambda,\xi)}\right]. \tag{69}$$

4. The stress-intensity factor history

When the normal stress on the crack plane $z = 0$ has been obtained, we now come to the determination of the dynamic stress-intensity factor for the fundamental problem. The stress-intensity factor in the Laplace transform domain can be expressed as

$$\overline{K}_{\mathrm{I}}^{\mathrm{F}}(\xi,s) = \lim_{x\to 0^+}[(2\pi x)^{1/2}\overline{\sigma}_+^{\mathrm{F}}(x,\xi,s)]. \tag{70}$$

From the Abel theorem concerning asymptotic properties of transforms and by virtue of Eq. (44), we get

$$\overline{K}_{\mathrm{I}}^{\mathrm{F}}(\xi,s) = \lim_{\eta\to+\infty}[(2s\eta)^{1/2}\Sigma_+(\xi,\eta,s)s^{-1}]. \tag{71}$$

Eq. (68) is substituted into Eq. (71) to give

$$\overline{K}_{\mathrm{I}}^{\mathrm{F}}(\xi,s) = \sqrt{\frac{2}{s}}\frac{\pi\rho k\exp(-sl\lambda)}{\lambda F_+(\lambda,\xi)}. \tag{72}$$

The inverse two-sided Laplace transform of Eq. (72) is

$$\widehat{K}_{\mathrm{I}}^{\mathrm{F}}(y,s) = \sqrt{\frac{2}{s}}\frac{s}{2\pi\mathrm{i}}\int_{\alpha_0-\mathrm{i}\infty}^{\alpha_0+\mathrm{i}\infty}\frac{\pi\rho k\exp\{-s[l\lambda(\xi)-\xi y]\}}{\lambda(\xi)F_+[\lambda(\xi),\xi]}\,\mathrm{d}\xi, \tag{73}$$

where $y > 0$ is assumed for the time being and α_0 is any real number between $-p_1$ and $+p_1$. Here, the inverse transform is carried out through the use of the Cagniard-de Hoop technique. Cagniard contours are introduced by setting $l\lambda(\xi) - \xi y = t$, which can be solved for ξ to yield

$$\xi_\pm = -\frac{yt}{y^2+l^2} \pm \frac{\mathrm{i}l}{y^2+l^2}\sqrt{t^2 - p_2^2v^2(y^2+l^2)}. \tag{74}$$

In the ξ plane, Eq. (74) describes a hyperbola which is denoted as $\Gamma_\pm$. When $t = t_0 = p_2v\sqrt{y^2+l^2}$, the imaginary part of $\xi_\pm$ vanishes and the vertex of the hyperbola $\Gamma_\pm$ is defined by

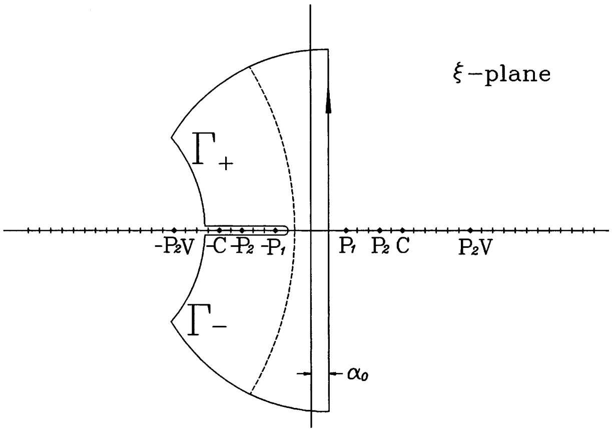

Fig. 2. The integration contour.

$$\xi_0 = -\frac{p_2 v y}{\sqrt{y^2 + l^2}}. \tag{75}$$

Now, we shift the ξ integration to the hyperbola paths $\Gamma_\pm$. The Γ_+ and Γ_-, together with the inversion path of ξ and two arcs of indefinitely large radius, form a closed contour as shown in Fig. 2. When $|\xi_0| < p_1$, the integrand of Eq. (73) is analytic inside and on this contour. Choose the appropriate branch of $\lambda(\xi)$ such that $\lambda(0) = p_2 v$. Then, according to Cauchy's integral theorem and Jordan's lemma, we obtain

$$\widehat{K}_{\mathrm{I}}^{\mathrm{F}}(y, s) = \pi\rho k \sqrt{\frac{2}{s}\frac{s}{\pi}}\,\mathrm{Im}\int_{t_0}^{\infty} \frac{\exp(-st)}{\lambda(\xi_+)F_+[\lambda(\xi_+), \xi_+]}\frac{\partial \xi_+}{\partial t}\,\mathrm{d}t. \tag{76}$$

From the convolution theorem for Laplace transform, we have

$$K_{\mathrm{I}}^{\mathrm{F}}(y, t) = \rho k \sqrt{\frac{2}{\pi}}\frac{\partial}{\partial t}\int_{t_0}^{t} \frac{G(y, \tau, v)}{\sqrt{t - \tau}}\,\mathrm{d}\tau, \tag{77}$$

where

$$G(y, \tau, v) = \mathrm{Im}\left\{ \frac{1}{\lambda(\xi_+)F_+[\lambda(\xi_+), \xi_+]}\frac{\partial \xi_+}{\partial \tau} \right\}, \tag{78}$$

and here the variable t in ξ_+ should be replaced by τ.

When $|\xi_0| > p_1$, an additional integration path from $-p_1$ to $-|\xi_0|$, which embraces the branch cut of $\lambda(\xi)F_+[\lambda(\xi), \xi]$, must be considered. In this case, Eq. (73) becomes

$$\widehat{K}_{\mathrm{I}}^{\mathrm{F}}(y, s) = \pi\rho k \sqrt{\frac{2}{s}\frac{s}{\pi}}\left\{ \mathrm{Im}\int_{t_0}^{\infty} \frac{\exp(-st)}{\lambda(\xi_+)F_+[\lambda(\xi_+), \xi_+]}\frac{\partial \xi_+}{\partial t}\,\mathrm{d}t - \mathrm{Im}\int_{p_1}^{|\xi_0|} \frac{\exp\{-s[l\lambda(\xi_1) + \xi_1 y]\}}{\lambda(\xi_1)F_+[\lambda(\xi_1), \xi_1]}\,\mathrm{d}\xi_1 \right\}. \tag{79}$$

Let $l\lambda(\xi_1) + \xi_1 y = \tau$, in which ξ_1 is in the range $p_1 \leqslant \xi_1 \leqslant |\xi_0|$. Then, we have $t_H \leqslant \tau \leqslant t_0$, where $t_H = p_1 y + l\sqrt{p_2^2 v^2 - p_1^2}$, and ξ_1 may be found to be

$$\xi_1 = \frac{y\tau}{y^2 + l^2} - \frac{l}{y^2 + l^2}\sqrt{p_2^2 v^2 (y^2 + l^2) - \tau^2}. \tag{80}$$

The stress-intensity factor in the time domain can be obtained as follows:

$$K_{\rm I}^{\rm F}(y,t)=\rho k\sqrt{\frac{2}{\pi}}\frac{\partial}{\partial t}\left[\int_{t_0}^{t}\frac{G(y,\tau,v)}{\sqrt{t-\tau}}{\rm d}\tau-\int_{t_H}^{t_0}\frac{G_1(y,\tau,v)}{\sqrt{t-\tau}}{\rm d}\tau\right] \tag{81}$$

with

$$G_1(y,\tau,v)={\rm Im}\left\{\frac{1}{\lambda(\xi_1)F_+[\lambda(\xi_1),\xi_1]}\frac{\partial\xi_1}{\partial\tau}\right\}. \tag{82}$$

Combining Eqs. (77) and (81), the dynamic stress-intensity factor for the fundamental problem may be expressed in the form

$$K_{\rm I}^{\rm F}(y,t)=\rho k\sqrt{\frac{2}{\pi}}\frac{\partial}{\partial t}\left[\int_{t_0}^{t}\frac{G(y,\tau,v)}{\sqrt{t-\tau}}{\rm d}\tau-\int_{t_H}^{t_0}\frac{G_1(y,\tau,v)}{\sqrt{t-\tau}}{\rm d}\tau H\left(vy-\frac{p_1}{p_2}\sqrt{y^2+l^2}\right)\right]. \tag{83}$$

With the fundamental solution and Lamb's solution at hand, it is now possible to construct the stress-intensity factor history for the case of impact point loading on the crack faces at $x=-l$, $y=z=0$. As described in the Section 2, this solution can be superimposed by two solutions, one is Lamb's problem with a concentrated force at $x=-l$, $y=z=0$, the other problem is that which cancels out the surface displacement for $z=0$, $x\geqslant 0$ of Lamb's problem. It is clear that the normal stress in Lamb's problem is not singular at $x=0$, $z=0$, so that the stress-intensity factor is determined by problem 2 only. From Eqs. (8)–(10), (26), (27) and (83), it is found that the stress-intensity factor is given by

$$\begin{aligned}K_{\rm I}(y,t)&=-\frac{F}{\pi^2\rho}\int_{p_1/p_2}^{\infty}K_{\rm I}^{\rm F}(y,t,v)f(v){\rm d}v\\&=-\frac{\sqrt{2}kF}{\pi^{5/2}}\int_{p_1/p_2}^{\infty}\frac{\partial}{\partial t}\left[\int_{t_0}^{t}\frac{G(y,\tau,v)}{\sqrt{t-\tau}}{\rm d}\tau-\int_{t_H}^{t_0}\frac{G_1(y,\tau,v)}{\sqrt{t-\tau}}{\rm d}\tau H\left(vy-\frac{p_1}{p_2}\sqrt{y^2+l^2}\right)\right]f(v){\rm d}v.\end{aligned} \tag{84}$$

Taking into account that $t_H\leqslant t_0\leqslant t$, we finally obtain

$$K_{\rm I}(y,t)=-\frac{\sqrt{2}kF}{\pi^{5/2}}\frac{\partial}{\partial t}\int_{p_1/p_2}^{v_0}\left[\int_{t_0}^{t}\frac{G(y,\tau,v)}{\sqrt{t-\tau}}{\rm d}\tau-\int_{t_H}^{t_0}\frac{G_1(y,\tau,v)}{\sqrt{t-\tau}}{\rm d}\tau H\left(vy-\frac{p_1}{p_2}\sqrt{y^2+l^2}\right)\right]f(v){\rm d}v, \tag{85}$$

where

$$v_0=\frac{t}{p_2\sqrt{y^2+l^2}}. \tag{86}$$

Although Eq. (85) is derived with the limitation $y>0$, it can be easily extended to the full range $-\infty<y<\infty$ by analytic continuation.

5. Results and discussions

We now discuss the properties of the dynamic stress-intensity factor history (85). It is observed that the first term represents the influence induced by the incident wave. The second term is due to the incident secondary wave produced by the first waves interacting with crack edge. The function $f(v)$ has a simple pole at c/p_2, and an immediate inference is that when the Rayleigh wave arrives, the stress-intensity factor is singular at this instant. Thereafter, $f(v)$ decays gradually as $t\to\infty$ and the dynamic stress-intensity factor approaches the value of static solution.

To make the physical meaning much clear, a numerical calculation of Eq. (85) is carried out for Poisson's material which is isotropic and for Beryl which is transversely isotropic.

Poisson's material: $a_1=a_2=3a_5,\quad a_3=2a_5,\quad a_4=a_5,\quad c=1.088/\sqrt{a_5}.$

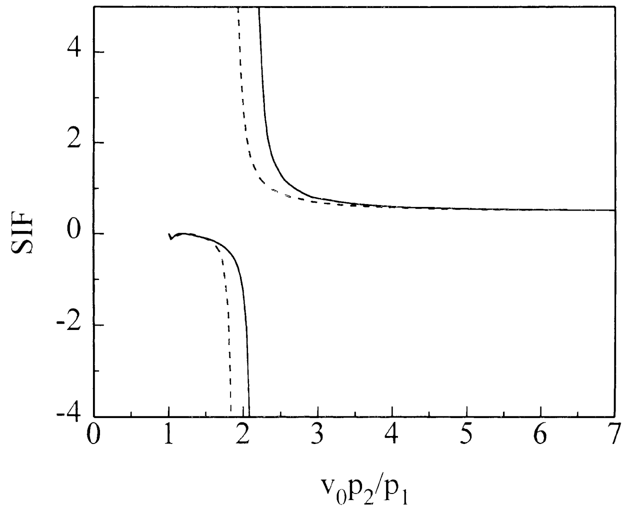

Fig. 3. The dynamic stress-intensity factor history $K_{\mathrm{I}}(y,t)$.

Beryl: $a_1 = 4.12484a_5$, $a_2 = 3.61802a_5$, $a_3 = 2.01199a_5$, $a_4 = 1.17363a_5$, $c = 1.04645/\sqrt{a_5}$.

Results for $y = l$ are shown in Fig. 3, with the dashed line for Poisson's material and the solid line for Beryl. In the figure, $\mathrm{SIF} = K_{\mathrm{I}}(y,t)((\pi l)^{3/2})/(\sqrt{2}F)$.

It is shown in Fig. 3 that before the arrival of the dilatational wave, the medium is completely at rest and the stress-intensity factor is zero. Upon sudden application of the normal point loads on the faces of the crack, the initial response is compressive and the crack faces tend to move towards each other. This is reflected by the stress-intensity factor being negative initially. This effect persists until the arrival of the Rayleigh wave at $t = (l^2 + y^2)^{1/2} \cdot c_{\mathrm{r}}^{-1}$. The stress-intensity factor is of the singularity $(v_0 - c/p_2)^{-1}$ at this instant. Thereafter, the transient stress-intensity factor decays gradually towards its equilibrium stress intensity factor which was obtained by Fabrikant et al. (1993).

This completes the analysis of a half plane crack in a transversely isotropic solid under the action of a pair of suddenly-applied normal point loads on the crack faces at a finite distance l away from the crack edge. An exact expression is derived for the mode I stress-intensity factor as function of time for any point along the crack edge. Unfortunately, the present approach is not valid to more general cases of anisotropy because Lamb's solution at this case cannot be expressed in the form of Eq. (10) and the usage of the Wiener–Hopf technique is then inhibited.

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References

Ang, W.T., 1988. A crack in an anisotropic layered material under the action of impact loading. J. Appl. Mech. 55, 120–125.
Ang, T., 1987. Transient response of a crack in an anisotropic strip. Acta Mechanica 70, 97–109.

Buchwald, V.T., 1961. Rayleigh waves in transversely isotropic media. Quart. J. Mech. Appl. Math. XIV, 293–317.

Dhawan, G.K., 1982a. Interaction of elastic waves by a Griffith crack in infinite transversely isotropic medium. Int. J. Fract. 19, 29–37.

Dhawan, G.K., 1982b. Interaction of SV-waves by a Griffith crack in infinite transversely isotropic medium. Int. J. Fract. 20, 103–110.

Erdelyi, A., 1954. Tables of Integral Transforms. McGraw-Hill, New York, pp. 277.

Eringen, A.C., Suhubi, E.S., 1975. Elastodynamics, vol. 2, Academic Press, New York.

Fabrikant, V.I., Rubin, B.S., Karapetian, E.N., 1993. Half-plane crack under normal load: a complete solution. J. Engng. Mech. 119, 2238–2251.

Freund, L.B., 1990. Dynamic Fracture Mechanics. Cambridge University Press, Cambridge.

Kassir, M.K., Bandyopadhyay, K.K., 1983. Impact response of a cracked orthotropic medium. J. Appl. Mech. 50, 630–636.

Kundu, T., Bostrom, A., 1991. Axisymmetric scattering of a plane longitudinal wave by a circular crack in a transversely isotropic solid. J. Appl. Mech. 58, 695–702.

Kundu, T., Bostrom, A., 1992. Elastic wave scattering by a circular crack in a transversely isotropic solid. Wave Motion 15, 285–300.

Kuo, A.Y., 1984a. Transient stress intensity factors of an interfacial crack between two dissimilar anisotropic half-spaces. Part 1. Othotropic material. J. Appl. Mech. 51, 71–76.

Kuo, A.Y., 1984b. Transient stress intensity factors of an interfacial crack between two dissimilar anisotropic half-spaces. Part 2. Fully anisotropic material. J. Appl. Mech. 51, 780–786.

Lin, W., Keer, L.M., 1989. Three-dimensional analysis of cracks in layered transversely isotropic media. Proc. R. Soc. Lond. A424, 307–322.

Lobanov, E.V., Novichkov, Y.N., 1981. Diffraction of SH-waves by an oblique crack in an orthotropic half-space. Soviet Appl. Mech. 17, 610–615.

Mandal, S.C., Ghosh, M.L., 1994. Interaction of elastic waves with a periodic array of coplanar Griffith cracks in an orthortropic elastic medium. Int. J. Engng. Sci. 32, 167–178.

Mattsson, J., Niklasson, A.J., Eriksson, A., 1997. 3D ultrasonic crack detection in anisotropic materials. Res. Nondestr. Eval. 9, 59–79.

Norris, A.N., Achenbach, J.D., 1984. Elastic wave diffraction by a semi-infinite crack in a transversely isotropic material. Quart. J. Mech. Appl. Math. 37, 565–580.

Ohyoshi, T., 1973. Effect of orthotropy on singular stresses produced near a crack tip by incident SH-waves. ZAMM 53, 409–411.

Shindo, Y., Nozaki, H., Higaki, H., 1986. Impact response of a finite crack in an orthotropic strip. Acta Mechanica 62, 87–104.

Shindo, Y., Higaki, H., Nozaki, H., 1992. Impact response of a single edge crack in an orthotropic strip. J. Appl. Mech. 59, s152–s157.

Tsai, Y.M., 1982. Penny-shaped crack in a transversely isotropic plate of finite thickness. Int. J. Fract. 20, 81–89.

Tsai, Y.M., 1988. Dynamic penny-shaped crack in a transversely isotropic material. Engng. Fract. Mech. 31, 977–984.

Xiaohua, Zhao, 1999. Three dimensional analyses of cracked rocks under transient loadings, Ph.D. Dissertation , Tongji University (in Chinese).

Xiaohua, Zhao, Huicai, Xie, 1999. The 3-D stress intensity factor for a half plane crack in a transversely isotropic solid due to the motion of loads on the crack faces. Acta Mechanica Sinica 15, 265–274.

Xiaohua, Zhao, Huicai, Xie, 2000. Elastodynamic analysis of a half plane crack in a transversely isotropic solid under 3-D transient loading. Acta Mechanica, in press.

Zhang, C.H., 1992. Elastodynamic analysis of a periodic array of model III crack in transversely isotropic solids. J. Appl. Mech. 59, 366–371.

Zhang, C.H., Gross, D., 1993. Interaction of antiplane cracks with elastic waves in transversely isotropic materials. Acta Mechanica 101, 231–247.